

# Modeling the Use of Self-Focused Beams to Overcome the Effects of Target Emissions in Advanced Hydrodynamic Radiography Machines

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## Modeling the use of Self-Focused Beams to Overcome the Effects of Target Emissions in Advanced Hydrodynamic Radiography Machines

Eugene Lauer

### 1-Introduction

In the machines being developed for advanced hydrodynamic radiography, an electron beam of several kA current and 20 Mev particle energy is focused to less than a millimeter diameter onto a high atomic number target to produce bremsstrahlung X-rays. Several pulses occur during a period of about  $2 \mu s$ . A plasma plume is predicted to move upstream from the target. If the final focus onto the target is in vacuum, then the plasma from an early pulse may neutralize the self-electric field of a later pulse causing over-focussing (1). Also positive ions may be accelerated upstream by the self-electric field of a beam focused onto a conducting target in vacuum (1,2). The ions neutralize part of the self-electric field and so cause a time varying change of focusing.

Several methods for overcoming these effects have been suggested:

1 The Livermore electron beam group has used the ETA-2 accelerator to show that a thin foil placed 2 to 3 cm upstream of the target will survive the beam pulse and block ions and plasma from moving upstream of the foil.

2 Low density targets and shorter pulses are being modeled by the Livermore group.

3 The final focusing magnet could be programmed to correct for the time dependent change caused by the plasma plume.

4 The beam pulses might be deflected transversely to enter a weak solenoidal magnetic field at a series of positions and then be compressed adiabatically by moving into stronger magnetic field. Each pulse would be in a separate cell which helps prevent the plasma from early pulses reaching the vacuum region of subsequent pulses (3).

5 Self focusing could be used just upstream of the target. A plasma would be used to completely neutralize the electric self-field (but retain the self magnetic

field). The massive beam electrons would expel plasma electrons and retain positive ions. The fractional neutralization  $f$  of the electric field would be 1 and couldn't be changed by the plume.

- a) One scheme for producing the self-focused beam is to use high density gas (a pressure of 10-100 Torr) in a chamber with a thin foil for the beam to enter. Direct ionization of gas by beam electrons produces the desired conditions in about a ns (4). Tests have been done at ETA-2. The main problem is that in order for the foil to survive the beam pulse, the beam radius at the foil must not be smaller than about 0.4 cm. This is large compared to the self-focused equilibrium radius, resulting in large beam envelope oscillations. The axial position of the minimum in radius is quite sensitive to changes in beam parameters. When the minimum changes its' axial position, there is a large change in beam radius at the target.
- b) In principle the beam could pass through a long ramp of electric neutralization fraction  $f$ , followed by a region in which  $f=1$  just upstream of the target under conditions so that the envelope would be in equilibrium in  $f=1$ . This idea is analyzed in the present report. The main problem is devising a scheme for producing the required ionized region in low density gas.

## 2-Beam Envelope Equation Model of the Formation of a Self- Focused, Fully Electrically Neutralized, Equilibrium, Relativistic Electron Beam

Our beam envelope equation is

$$\frac{d^2R}{dz^2} = \frac{\epsilon^2}{R^3} - \frac{(f[z]-1/\gamma^2)I}{(1-1/\gamma^2)\beta\gamma 17.05R} \quad 1$$

$R$  is the rms beam radius (cm),  $z$  is the axial coordinate (cm),  $\epsilon$  is the rms emittance (radian cm),  $f$  is the fractional neutralization of the radial electric field of the beam,  $I$  is the beam current (kA), 17.05 has units of kA,  $\gamma$  is the beam electron total energy in rest mass units

$$\gamma = \frac{U + .5110}{.5110} \quad 2$$

where  $U$  is the beam electron kinetic energy in Mev.

$$\beta = \sqrt{1-1/\gamma^2} \quad 3$$

With  $f=1$ , the equilibrium radius of the beam is

$$R_{eq} = \epsilon \sqrt{\beta\gamma 17.05/I} \quad 4$$

and the corresponding beam electron density is

$$n_b = \frac{100I^2}{\pi e \beta^2 \gamma 17.05 \epsilon^2}$$

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where e is 4.803 esu.

Table 1 lists the parameters for ETA 2 and DARHT

Table 1-PARAMETERS FOR ETA 2 AND DARHT

	ETA 2	DARHT
U(Mev)	6	20
$I_b$ (kA)	1.8	2
$\epsilon$ ( $10^{-3}$ rad cm)	4	1.5
$\gamma$	12.7417	40.1389
$\beta$	0.996915	0.999690
$R_{eq}$ (cm)	.043876	.027743
$n_b$ ( $10^{13} cm^{-3}$ )	6.216	17.23

Fig 1 shows examples of linear f- ramps of several lengths, followed by a region of f=1. The Mathematica numerical program was used to solve Eq 1 with the two conditions that  $R = R_{eq}$  and  $R' = 0$  in the f=1 region. Fig 2 shows ETA 2 beam envelopes entering the ramps of Fig 1 with the correct combination of amplitude and slope so as to end up in equilibrium in the f=1 region. Fig 3 shows DARHT envelopes which are in equilibrium in f=1 for the ramps of Fig 1. Fig 4 shows three quadratic f-ramps, and Fig 5 shows the corresponding ETA envelopes.

These calculations show that if the required ionized region can be created, then the beam can be injected so as to end up in equilibrium in the f=1 region.

### 3-Methods of creating the required gas ionization

It might be necessary to re-create the ionized region before each beam pulse because the positive ions that were confined by the beam current would accelerate radially outward after the beam current ends. On the other hand, if electrons are available, they might accelerate inward neutralizing the ions and preserving the ionization. The ionization must be created with a gas density of only about  $10^{14} cm^{-3}$  in order to avoid excessive time dependent increase of f in the ramp by direct beam ionization.

#### a-Laser ionization

Gas would be leaked at a steady rate into a chamber located just upstream of the target, and pumped out through a restricting tube with the axis aligned with the beam axis at the upstream end of the chamber. The gas molecule density in the f=1 chamber would be about equal to the equilibrium beam density ( a pressure of about  $2 \times 10^{-3}$  Torr for ETA), and would fall smoothly to zero at the upstream end of the tube. The laser beam would be brought in at a bend in the beam line and directed along the axis through the tube and chamber. The laser pulse would ionize most of the molecules in its' path creating the ramp and f=1 region. At a laser intensity greater than about  $10^{14}$  watts/cm<sup>2</sup>, short pulse lasers have been shown to ionize noble gases very efficiently(5). Lasers exist which might work(6), but they cost several hundred thousand dollars at the present time. Laser guiding was used on the ATA beam (7). The benzene ion density was only a few percent of what is required in the present case, also we only need a beam path length of a few 10s of cm.

#### b-Electron diode with gas ionization

A planar electron diode containing gas is analyzed in a separate report(8). When the external voltage is switched onto the electrodes at time zero, the ion density is zero and the electron current density has the Child-Langmuir value. The electrons are accelerated and ionize gas and the resulting positive ions neutralize part of the electron space charge. Then the electron current density starting from the cathode increases and the ionization rate increases etc. The analysis indicates that the required ionization density might be reached in a fraction of a microsecond. The electrodes could be closely spaced straps with the beam centered between them. The gas density would be uniform and the ramp could be created by smoothly bending the straps farther apart at the upstream end.

#### 4-Time dependent departure from equilibrium

The injection may be optimized so that the envelope is in equilibrium in f=1 at some particular time, eg. the center of the beam current pulse. Then any time dependent variation in parameters can cause a lack of equilibrium at the head or tail of the pulse.

#### a-Upstream motion of positive ions due to Ez field for a beam in a long f-ramp.

First we calculate the Ez field in this situation We use Poisson's equation in cylindrical coordinates with axial symmetry

$$\frac{1}{r} \frac{\partial(rE_r)}{\partial r} + \frac{\partial E_z}{\partial z} = 4\pi\rho$$

6

(Gaussian units)

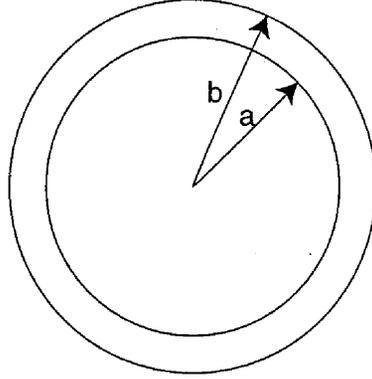


Fig 1 Cross section of beam and ion channel

The charge density  $\rho$  has contributions from the beam electrons  $\rho_b$ , and from the unneutralized positive ions  $\rho_i$ .  $\rho_i$  is uniform inside the circle with radius  $b$  (Fig 1). The net charge inside  $b$  is zero, and  $\mathbf{E}_r = \mathbf{0}$  at  $r=b$ . For large  $f$ ,  $b \ll r_w$  (the wall radius). For small  $f$ ,  $b > r_w$  The beam radius  $a$  is modeled as independent of  $z$ .

$$\rho = \rho_b + \rho_i \quad 7$$

$$\rho_b = -\frac{I_b}{\beta c \pi a^2}, r \leq a$$

where  $I_b$  is the magnitude of the beam current. 8

$$\rho_i = -f\rho_b, r \leq b \text{ (or } r_w \text{ whichever is smaller)} \quad 9$$

where  $f$  is the fractional neutralization.

$$\rho_b a^2 = -\rho_i b^2 = f\rho_b b^2$$

$$a^2 = fb^2 \quad 10$$

We guess that  $\frac{\partial E_z}{\partial z}$  can be neglected in Eq 6 and solve for  $\mathbf{E}_r$  in the two regions.

For  $r \leq a$ ,

$$\frac{1}{r} \frac{d(rE_r)}{dr} = \frac{4I_b}{\beta ca^2} (f-1)$$

$$\int d(rE_r) = \frac{4I_b}{\beta ca^2} (f-1) \int r dr$$

$$rE_r = \frac{2I_b}{\beta ca^2} (f-1)r^2 + \text{const}$$

At  $r=0$ ,  $\mathbf{E}_r = \mathbf{0}$ , so  $\text{const} = 0$ .

$$E_r = \frac{2I_b}{\beta ca^2} (f-1)r, r \leq a$$

11

For  $r \geq a$ ,

$$\int d(rE_r) = \frac{4I_b}{\beta ca^2} f \int r dr$$

$$rE_r = \frac{2I_b}{\beta ca^2} fr^2 + \text{const}$$

$$\text{At } r=a, E_r = \frac{2I_b}{\beta ca^2} (f-1)a^2$$

$$\frac{2I_b}{\beta ca^2} (f-1)a^2 = \frac{2I_b}{\beta ca^2} fa^2 + \text{const}$$

$$\text{const} = -\frac{2I_b}{\beta ca^2} a^2$$

$$rE_r = \frac{2I_b}{\beta ca^2} (fr^2 - a^2)$$

$$E_r = \frac{2I_b}{\beta ca^2} \left( fr - \frac{a^2}{r} \right), a \leq r \leq b \text{ (or } \mathbf{r}_w)$$

12

Assuming  $\mathbf{E}_z \ll \mathbf{E}_r$ , we integrate  $\mathbf{E}_r$  in  $r$  at constant  $z$  to get the potential,  $\Phi$  on axis, ( $\Phi = 0$  at  $b$  or  $\mathbf{r}_w$ ).

$$\Phi = \frac{2I_b}{\beta ca^2} \left[ (f-1) \frac{a^2}{2} - a^2 \ln \frac{b}{a} + \frac{f}{2} (b^2 - a^2) \right], b(r_w)$$

$$\Phi = \frac{2I_b}{\beta c a^2} \left[ -\frac{a^2}{2} - a^2 \ln \frac{b}{a} + \frac{f b^2}{2} \right], b < r_w$$

Using Eq 10

$$\Phi = -\frac{2I_b}{\beta c} \ln(f^{-1/2}), b < r_w \quad 13$$

$$\Phi = \frac{2I_b}{\beta c a^2} \left[ (f-1) \frac{a^2}{2} - a^2 \ln \frac{r_w}{a} + \frac{f}{2} (r_w^2 - a^2) \right], b > r_w$$

$$\Phi = \frac{2I_b}{\beta c} \left[ -\frac{1}{2} - \ln \frac{r_w}{a} + f \frac{r_w^2}{2a^2} \right], b > r_w \quad 14$$

Let  $r_w = 2a$ , then at  $b = r_w, f = 1/4$ . We now differentiate the z-dependent terms in  $\Phi$  for two ramp shapes.

Linear ramp,  $f = z/L$ ,

For  $b \leq r_w$ ,

$$\begin{aligned} -\frac{d \ln(z/L)^{-1/2}}{dz} &= (z/L)^{1/2} (-) \frac{1}{2} (z/L)^{-3/2} \frac{1}{L} \\ &= -(z/L)^{-1} \frac{1}{2L} \rightarrow -\frac{2}{L} \ln(z/L) = 1/4 \\ &\rightarrow -\frac{1}{2L} \ln(z/L) = 1 \end{aligned}$$

For  $b \geq r_w$

$$-\frac{d}{dz} (-r_w^2 z / 2a^2 L) = r_w^2 / 2a^2 L = -\frac{2}{L}$$

Quadratic ramp,  $f = (z/L)^2$

$b \leq r_w$

$$\begin{aligned}
-\frac{d}{dz} \ln(z/L)^{-1} &= (z/L)(-)(z/L)^{-2} \frac{1}{L} \\
&= -(z/L)^{-1} \frac{1}{L} \rightarrow -\frac{2}{L} at(z/L) = 1/2
\end{aligned}$$

$b \geq r_w$

$$\begin{aligned}
-\frac{d}{dz} \left[ -\frac{r_w^2}{2a^2} (z/L)^2 \right] &= -\frac{r_w}{a^2} (z/L) \frac{1}{L} \\
&= -4(z/L) \frac{1}{L} \rightarrow -\frac{2}{L} at(z/L) = 1/2 \\
&\rightarrow 0 at(z/L) = 0
\end{aligned}$$

The maximum magnitude of  $\mathbf{E}_z$  for these two ramps is

$$\mathbf{E}_z = \frac{4I_b}{\beta c L} \quad 15$$

Physically this is approximately twice the potential on axis with  $f=0$ , divided by  $L$ . This may be contrasted with the case of an unneutralized beam with radius  $a$  incident on a conducting surface where  $\mathbf{E}_z$  has a similar expression but with  $L$  replaced by  $a$ .

The assumptions that  $\partial \mathbf{E}_z / \partial z$  and  $\mathbf{E}_z$  can be neglected in deriving the potential is justified for  $L \gg a$ .

We now calculate the maximum upstream motion of ions in the  $\mathbf{E}_z$  field of a linear or quadratic  $f$ -ramp during one beam pulse.

$$\frac{d^2 z}{dt^2} = \frac{e \mathbf{E}_z}{M} \quad 16$$

Integrating,

$$\Delta z = \frac{e \mathbf{E}_z}{2M} t^2 \quad 17$$

Let  $I_b = 1.8 \text{ kA}$  or  $1.8 * 3 * 10^{12} \text{ esu}$ ,  $L = 20 \text{ cm}$ ,  $\beta = 1$ , then  $E_z = 36 \text{ esu}$ ,

$t = 5 * 10^{-8} \text{ sec}$ ,  $M = 40 / (6 * 10^{-23}) \text{ gm (for Ar}^+)$

Then

<i>ION</i>	$\Delta z(cm)$
<i>Ar<sup>+</sup></i>	.32
<i>Kr<sup>+</sup></i>	.15

Fig 7 shows the ETA-2 beam envelope with the same entry conditions as for the 20 cm ramp of Fig 2, but with the ramp moved upstream by 0.16 cm. This models the departure from equilibrium at the tail of the pulse when the injection is optimized part way through the pulse. Small amplitude oscillations of the envelope radius are evident in the f=1 region. The theoretical wavelength is

$$\lambda = \frac{\sqrt{2}\pi\epsilon(1-1/\gamma^2)\beta\gamma 17.05(kA)}{(f-1/\gamma^2)I(kA)} cm \quad 18$$

For ETA-2 parameters,  $\lambda = 2.1$  cm.

b- Linear ramp steepened due to gas ionization by beam electrons.

The rate of ionization is

$$\frac{dn_i}{dt} = n_b n_g \sigma \beta c \quad 19$$

where  $n_g$  is the gas density ( $cm^{-3}$ ) and  $\sigma$  is the cross section. The fractional change in f that occurs in time t is

$$\Delta = n_g \sigma \beta c t \quad 20$$

For ETA-2 and Argon gas,  $\sigma = 1.04 \times 10^{-18} cm^2$  (9). Using  $n_g = 6.2 \times 10^{13} cm^{-3}$  and  $t=25$  ns,  $\Delta = 0.048$  If the injection is optimized at the center of the 50 ns beam pulse, then at the tail of the pulse a linear f-ramp that was originally 40 cm long would be shortened by 1.83 cm. This case is shown on Fig 8.

c-time variation of beam current.

At the head and tail of the ETA-2 beam current pulse, the current is about 5% less than at the center. This effect is modeled on Fig 9.

All of the time dependent departures from equilibrium modeled in this section are acceptably small.

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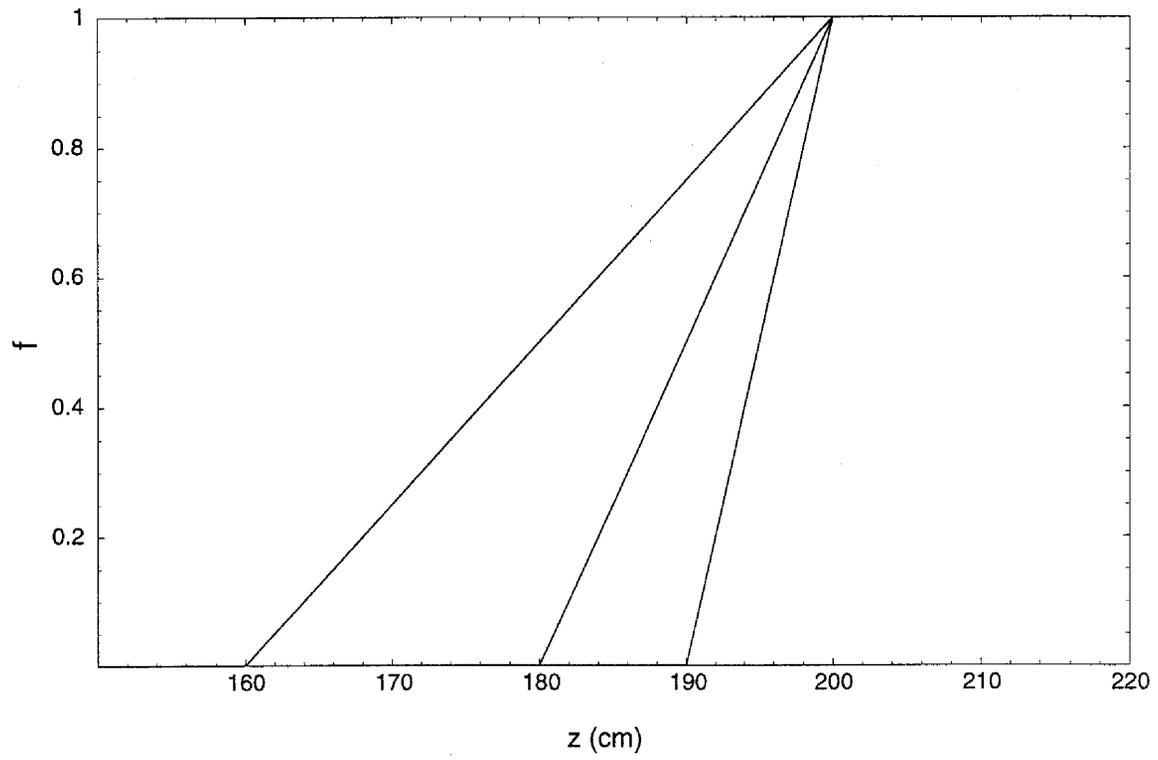


Fig 2 LINEAR f-RAMPS 10, 20, 40 cm LONG

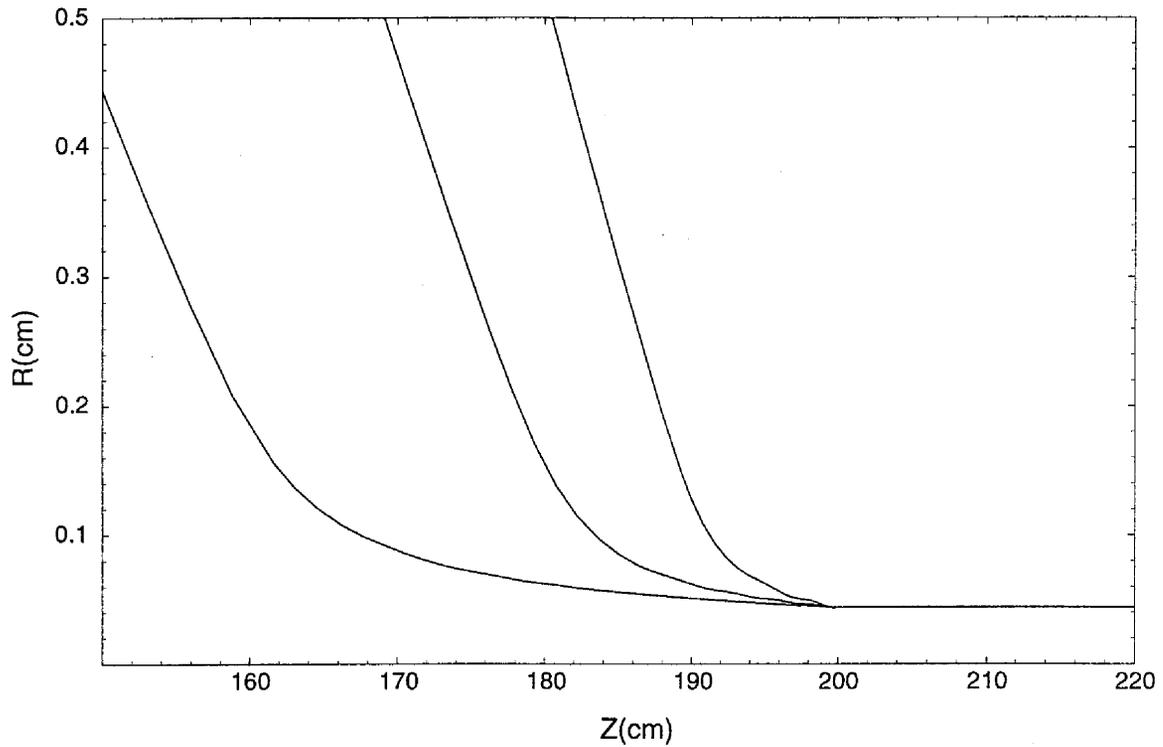


Fig. 3-. ETA-2, BEAM ENVELOPE IN EQUILIBRIUM IN  $f=1$  AFTER PASSING THROUGH 10, 20, 40 cm LINEAR f-RAMPS

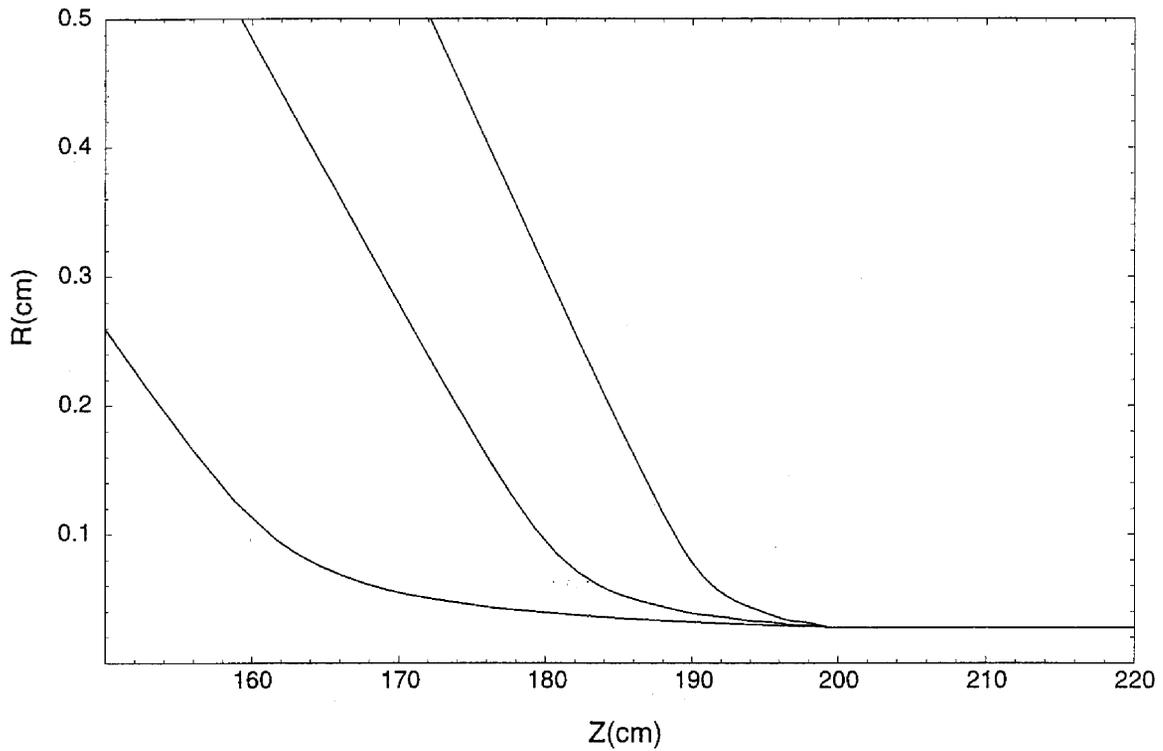


Fig 4- DARHT, BEAM ENVELOPE IN EQUILIBRIUM IN  $f=1$  AFTER PASSING THROUGH 10, 20, 40 cm LINEAR f-RAMPS

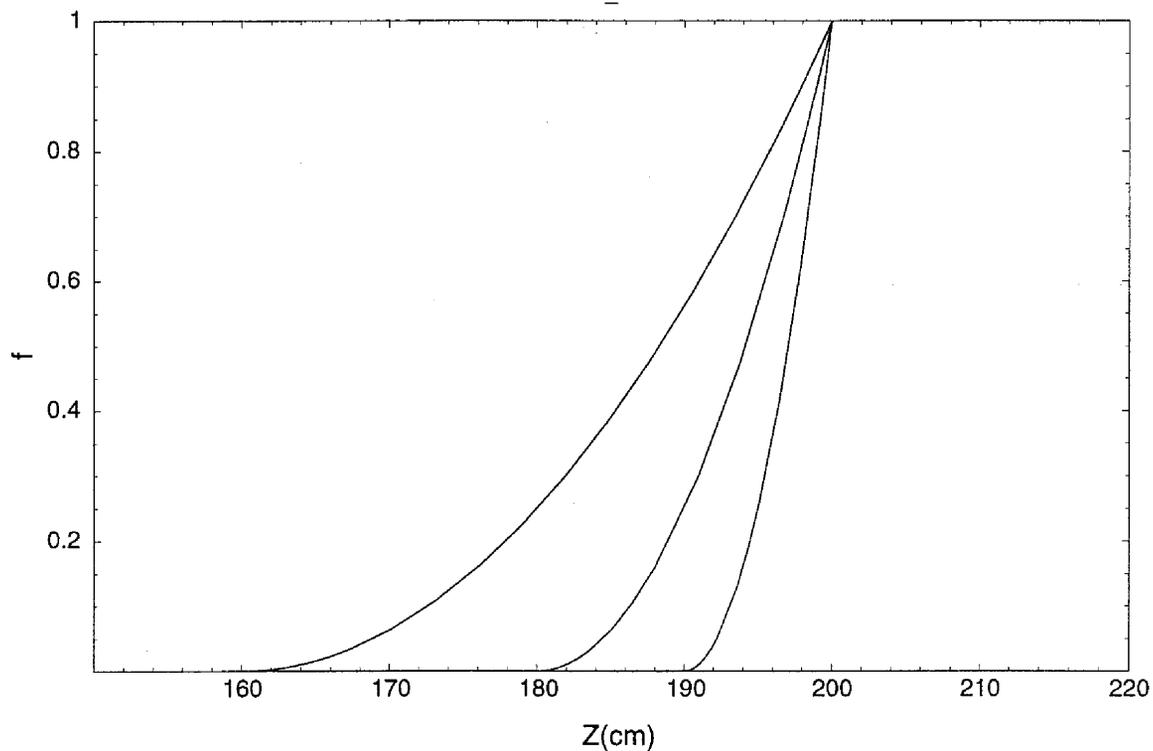


Fig 5-QUADRATIC f-RAMPS 10, 20, 40 cm LONG

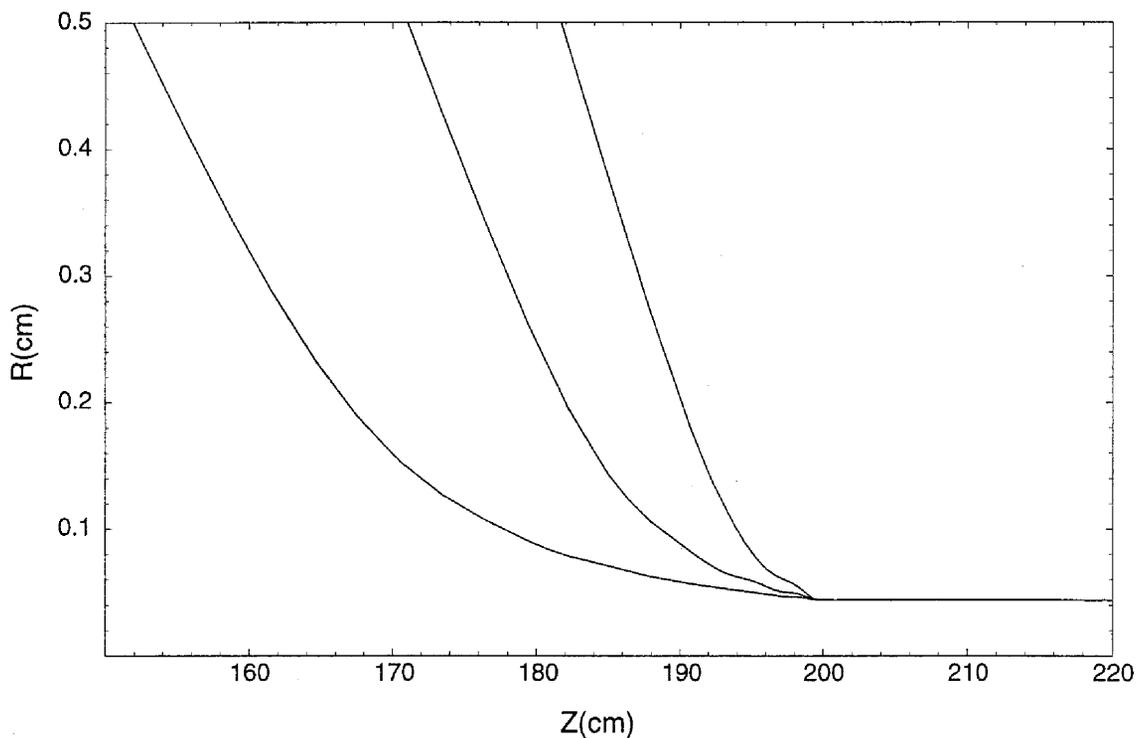


Fig 6-ETA-2 BEAM ENVELOPE IN EQUILIBRIUM IN  $f=1$  AFTER PASSING THROUGH 10, 20, 40 cm QUADRATIC f-RAMPS

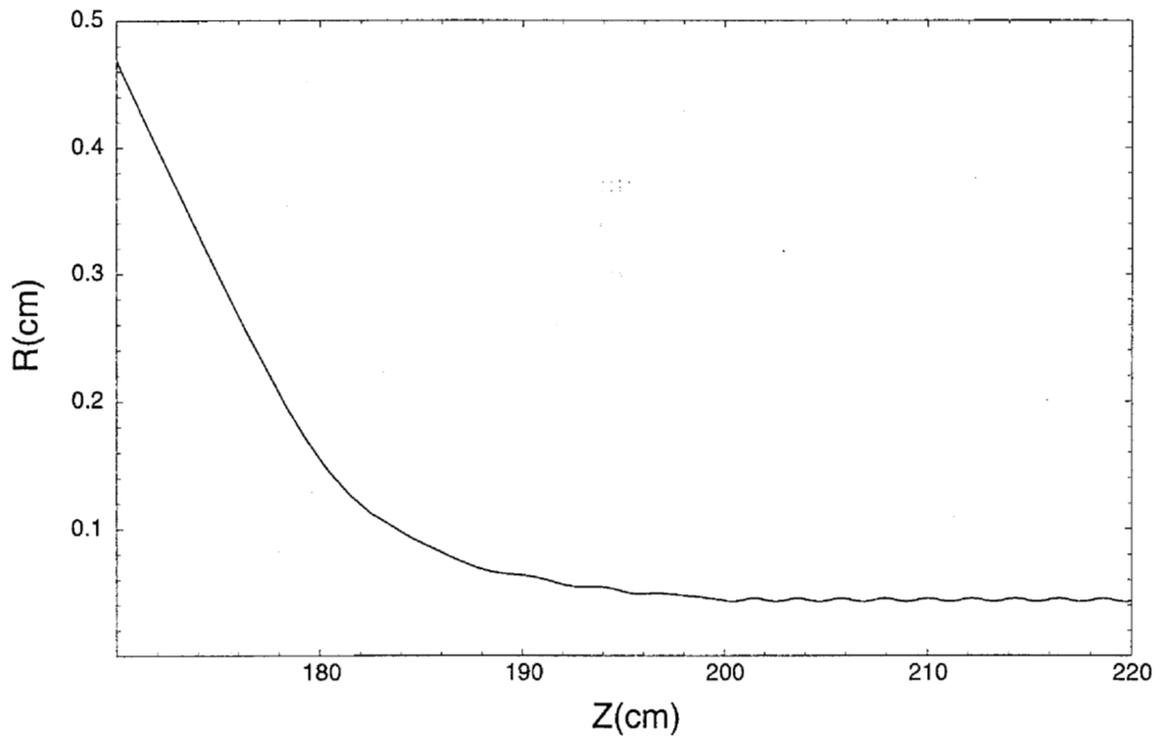


Fig 7-ETA-2,  $Ar^+$ , 20 cm LINEAR RAMP MOVED UPSTREAM BY  $E_z$ , TAIL OF PULSE

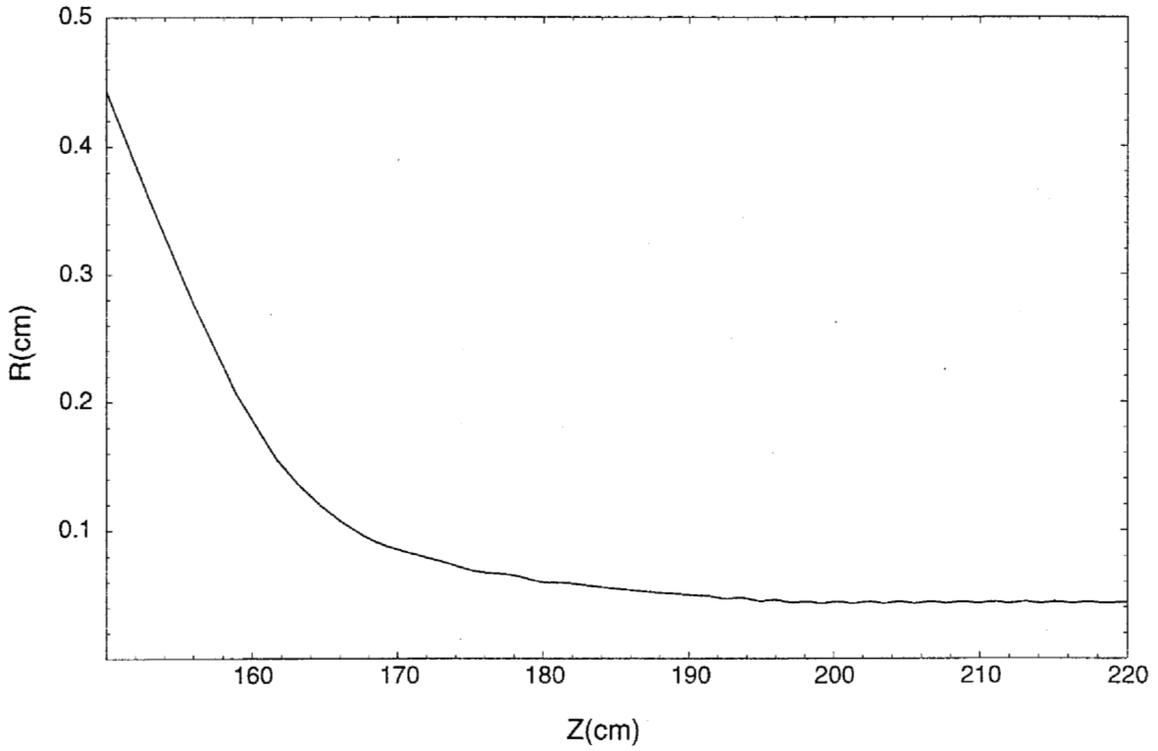


Fig 8-ETA-2, BEAM ENVELOPE WITH 40 cm LINEAR f-RAMP SHORTENED BY 1.83 cm

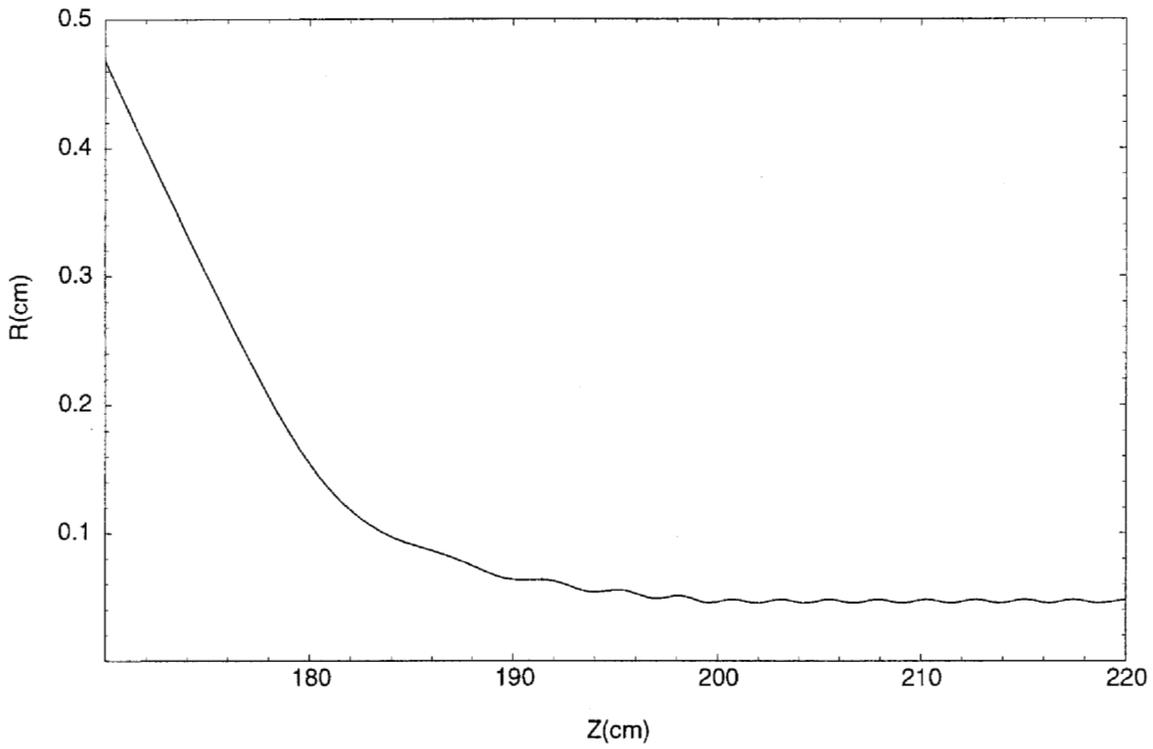


Fig 9-ETA-2, BEAM CURRENT DECREASED BY 5%



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